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AIR FORCE MATERIALS LAB WRIGHT-PATTERSON AFB OHIO
STATISTICAL ANALYSIS OF ONE HUNDRED AND TWELVE GROUPS OF FATIGUE--ETC(U)
APR 78 W WEIBULL
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**STATISTICAL ANALYSIS OF ONE HUNDRED AND
TWELVE GROUPS OF FATIGUE PERFORMANCE DATA:
TESTING THE HOMOGENEITY OF THE SAMPLES**

*AVENUE d'ALBIGNY, 9 BIS
74000 ANNECY
FRANCE*

APRIL 1978

TECHNICAL REPORT AFML-TR-78-28

Final Report for the Period 15 January 1974 - 15 February 1974

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AIR FORCE MATERIALS LABORATORY
AIR FORCE WRIGHT AERONAUTICAL LABORATORIES
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO 45433

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FOREWORD

The research work reported herein was conducted by Prof. Dr. Waloddi Weibull, Avenue d'Albigny, 9 bis, 74000 Annecy, France, under USAF Contract No. F44620-73-C-0066. This contract, which was initiated under Project No. 7351, "Metallic Materials", Task 735106, "Behavior of Metals", was administered by the European Office of Aerospace Research. The work was monitored by the Metals and Ceramics Division, Air Force Materials Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio 45433, under the direction of Mr. W. J. Trapp, AFML/LL.

This report covers work conducted during the period 15 January 1974 to 15 February 1974. The manuscript was submitted by the author for publication in March 1974.

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1. INTRODUCTION

The first step in the procedure of statistically analysing a given group of test data should always be to state the homogeneity of the group, that is, to test the hypothesis H_0 that all the elements of the sample belong to one and the same population. The importance of this advice may be emphasized by the fact that 64 of the 112 here examined samples were rejected due to lacking homogeneity.

Several such tests have been proposed. The scope of the present investigation was to study the discrimination power of three of them by applying them to a large number of groups of fatigue performance data taken from a very extensive collection performed at the Boeing Company. Data for 2,000 groups representing 11,000 specimens have been collected and presented by Whittaker and Besuner (1). All complete samples of sizes from ten to twenty, being in total 112 samples representing 2,095 specimens, were taken out and examined. The numbers of kilocycles to failure are listed in Table 1, which also contains an 11-digit data coding number, cataloging the variables of specimen thickness, material, grain direction, type of structures, type of specimen, finish, type of loading and type of failure. The data coding system is presented in Table 3. A list of references to the Boeing items is given in Table 4.

The three methods of testing homogeneity, which have been studied, are denoted by MLE, OMLE and TI and are described in the following sections.

2. THE MLE METHOD

The symbol MLE stands for "maximum likelihood estimation". This classical method is known to have maximum efficiency. It has recently been shown (2) to have also maximum reliability.

In the case of three unknown parameters (m, x_u, x_o) , the estimate is any point $(m, \hat{x}_u, \hat{x}_o)$ in the parameter space at which the likelihood function $L(m, x_u, x_o)$ of the sample (x_i) is a maximum. This point is obtained as a solution of the likelihood equations

$$\partial L / \partial m = 0, \quad \partial L / \partial x_u = 0, \quad \partial L / \partial x_o = 0 \quad (1)$$

For physical and technological reasons, the parameter space has to be restricted by the conditions

$$m \geq 1.0, \quad 0 \leq x_u \leq \min x_i \quad (2)$$

If no solution of the likelihood equations (1) exists which satisfies the conditions (2), then the hypothesis H_0 will be rejected.

For the numerical computation of the estimates it has for several reasons been found more convenient, instead of solving for the conditions in (2), to compute the value of

$$L^*(m, x_u) = \log L(m, x_u, \hat{x}_0) \quad (3)$$

Where \hat{x}_0 is obtained from the likelihood equation $\partial \log L / \partial x = 0$, for an appropriate set of (m, x_u) and to select the particular pair (\hat{m}, \hat{x}_u) which maximizes L^* .

This procedure is performed by use of Program 6/73 which computes the values of L^* for the ten values $\alpha = 1/m = 0.1(0.1)1.0$ and the twelve values $x_u = x_1 \cdot (i/10)$, ($i = 0(1)10$) and $x_u = 0.99x_1$, and then, by interpolation, the pair (\hat{m}, \hat{x}_u) which maximizes L^* . The computing time for a sample of size $N = 10$ is about 0.6 seconds per sample by use of the computer IBM 360, M 75. It has been shown by McCool (3) and by Antle and Klimko (4) that, for a given x_u , this estimate always exists and is unique.

The hypothesis H_0 is rejected if $\alpha > 1.0$, that is, $m < 1.0$ or $x_u < 0$.

The results are presented in Table 2 under the caption MLE, Program 6/73.

It may be of interest to compare these estimates \hat{m} with those presented by Whittaker and Besuner (1) which are obtained under the assumption that $x_u = 0$, and listed also in Table 2 under the caption Boeing, m. This assumption is believed to be unrealistic and explains the great deviations from the estimates presented under the caption MLE Program 6/73. The numbers under the caption Boeing, nr, indicate the number of order statistics actually used for computation of the Boeing estimates \hat{m} . The sample size reduction is due to the omission of high-time outliers and may be taken as an indication of lacking homogeneity of the sample.

3. THE OMLE METHOD

This method is based on the quotients s_i of the order statistics x_i , defined by

$$s_i = (x_i - x_u) / (x_N - x_u) \quad (4)$$

It is evident that these quotients are independent of the parameter

x_o . Formulas for the density functions $f(s_i; m, x_u)$ have been developed. Estimates are taken for which the particular values (\hat{m}, \hat{x}_u) maximize the product of the density functions $f(s_i; m, x_u)$. (It should be noted that this product is not strictly identical with the likelihood function of the sample (s_i)).

The numerical calculations are performed by use of the Program 14/73. The computing time for a sample of size $N = 10$ is about 0.3 seconds per sample by use of the computer IBM 360, M 75. The hypothesis H_o is rejected, if the estimates do not satisfy the conditions in (2). The results are presented in Table 2 under the caption OMLE Program 14/73.

4. THE TI METHOD

This method is based on the concept of pseudo-standardized variables, as fully described in an AFML Technical Report (5). The variable

$$t_i = (x_i - x_1) / (x_N - x_1) \quad (5)$$

is independent of the parameters x_u and x_o . Its percentiles of the orders 5% and 95% have been determined by use of Monte-Carlo studies. These values are used as the limits of the acceptance regions. If any one of the order statistics t_i falls outside this region, the hypothesis H_o is rejected. The results are presented in Table 2 under the caption TI, $m \geq 1.0$.

This method may also be used for testing the hypothesis that the sampling is drawn from a normal population. The results are given in the column with the caption TI a><. The three-digit numbers indicate the number of order statistics falling within, above and below the acceptance region. Consequently, only samples having a number 800 or 18.0.0 are accepted. (The regions for $N = 11, 12, 13, 14, 15$ are not known).

5. RESULTS AND CONCLUSIONS

The estimates of MLE and OMLE and the coding numbers of TI, $m \geq 1.0$ are presented in Table 2, where the column with the caption TI a>< corresponds to the hypothesis that the sample is drawn from a normal population.

A summary of the three tests is presented in Table 5, which indicates that the MLE test is sharper than the OMLE and TI tests.

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4. Antle, C. E. and Klimko, L. A., "Choice of Model for Reliability Studies and Related Topics II", RRL-73-0121, October 1973.
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TABLE 1. LISTED NUMBERS OF KILOCYCLES TO FAILURE

Ref. 1 Data Coding Number: 040 10 86 00 50

Item	Number of kilocycles to failure									
459	228	239	240	254	260	262	277	300	309	359

Ref. 3 Data Coding Number: 300 12 85 58 30

651	179	208	219	266	298	314	401	430	763	929
652	319	374	504	600	665	665	776	776	2993	3844
930	902	979	1738	1876	3005	6902	7449	18223	26681	42229
653	490	491	803	888	1021	1034	1160	1249	1748	1809
653	3643	-	-	-	-	-	-	-	-	-

Ref. 3 Data Coding Number: 300 12 86 58 30

44	70	76	80	81	86	108	142	144	282	3318
45	82	94	131	131	207	229	296	349	721	1160
46	7.1	7.4	7.8	8.0	8.4	8.7	9.0	9.4	9.6	9.9
47	15.4	15.9	16.0	16.1	16.3	16.4	17.1	17.6	17.7	19.2
48	40	44	46	51	52	57	59	63	67	269
49	121	171	205	446	480	592	642	877	1916	18499
50	41	182	210	303	405	423	643	722	739	763
51	17	17	20	28	29	33	36	36	36	41
52	29	31	36	44	47	52	71	399	445	689
53	224	253	263	328	423	641	690	708	782	842
54	799	840	846	867	984	1020	1156	1276	1383	1388
323	340	375	766	967	1388	1515	4611	4694	12838	22918
324	11348	11785	11924	12560	15223	18995	19132	23875	27558	32273
327	242	1051	1107	1122	1552	1565	1880	2212	3732	6274
328	1289	1296	1772	2429	3033	3074	9625	12840	13434	71645
331	735	1116	1132	1430	1439	1445	1479	1490	1766	2449
549	743.4	865.9	968.3	1106.4	1109.7	1110.1	1326.3	1337.9	1438.7	1906.7
548	1554	1669	1676	1773	1781	1790	1893	1906	1989	2005
548	2019	2117	2329	2450	-	-	-	-	-	-
407	168.6	175.5	181.1	198.2	200.7	203.9	208.6	213.3	214.1	220.3
407	222.2	225.5	247.1	247.9	280.1	-	-	-	-	-

TABLE 1. (CONTINUED)

Ref. 4	Data Coding Number: 300 06 85 58 30									
Item	Number of kilocycles to failure									
3650	15	18	18	18	20	20	22	24	26	28
3651	89	114	121	135	141	146	159	161	163	171
3653	334	652	689	782	885	1056	1069	1110	1114	1437
3654	643	714	988	1084	1152	1465	1563	1771	1941	2481
3655	317	597	650	734	745	835	840	846	899	1224
3656	378	391	496	513	600	687	1013	1074	1161	1483
3657	606	665	667	824	842	996	1057	1133	1943	2505
3658	141	k42	156	168	169	328	383	498	505	698
3659	217	252	258	306	328	368	391	439	468	659
3660	185	252	263	353	405	543	567	587	651	704
3661	210	225	259	263	275	317	339	516	811	838
3662	110	113	120	138	254	272	300	320	422	434
3663	223	242	261	272	276	316	323	323	378	386
3922	2224	2464	2482	2706	2766	2882	3098	3650	4162	4223
3923	12090	13093	13206	13631	16536	17867	22119	23178	29890	50278
3927	3312	5200	6594	9014	12231	19260	19670	23186	24456	24574
3928	18540	18783	22354	28510	35247	39360	40983	47717	56163	64429
3929	597	1330	1704	2028	2879	3138	3302	4026	4666	6787
3930	2668	3442	3903	4334	4664	5338	5447	5614	5656	5864
3931	876	1156	1451	1806	2021	2061	2248	2569	2899	3929
3932	1190	1360	1389	1555	2057	2632	5830	5920	7684	13731
3933	3560	3832	6092	6785	7246	14090	15438	21517	28810	30764
3934	1526	1673	1997	2100	2599	2599	2828	3610	4226	5343
3935	1778	3092	3121	3628	4545	5208	7605	8118	9870	11716
3936	1924	4124	5231	5466	5592	6210	6882	7680	8076	10114
3937	481	740	833	1224	2041	2414	3441	4260	4371	5776
3938	540	1255	1302	1442	1497	1801	1879	2197	2679	4097
3939	772	860	928	932	1035	1049	1051	2316	2340	2403
3940	938	969	1592	1845	1875	1915	1940	2235	3008	6758
3652	377	425	445	483	523	606	1032	1167	1258	2002
"	2616	2616	-	-	-	-	-	-	-	-
3924	19082	33740	63674	89681	90265	100725	106542	117843	118000	270169
"	317912	-	-	-	-	-	-	-	-	-
3926	1158	1304	1505	1544	1555	2225	2467	2709	3139	3835
"	4972	-	-	-	-	-	-	-	-	-
3920	393	699	776	786	792	886	1187	1190	1220	1349
"	1370	1474	1510	1595	1693	1726	1792	1796	2242	4700
3921	2292	3993	4013	4130	4570	4801	5093	5170	5343	6257
"	6521	6957	7331	7422	7576	8236	8239	11864	12493	17528

TABLE 1. (CONTINUED)

Ref. 7 Data Coding Number: 031 02 02 10 10

Item	Number of kilocycles to failure									
4054	26.5	33.2	36.7	37.3	38.1	39.5	41.0	41.8	43.8	47.9
"	52.6	53.1	-	-	-	-	-	-	-	-
4055	23.5	41.8	43.3	43.3	47.2	53.1	58.5	61.4	69.9	83.1
"	84.7	85.3	-	-	-	-	-	-	-	-
4056	45.7	52.3	75.0	75.1	88.4	101.9	115.2	118.8	121.7	127.5
"	148.3	201.4	-	-	-	-	-	-	-	-
4057	126.6	175.1	207.0	241.5	269.5	275.7	299.1	311.1	367.6	460.9
"	523.4	526.1	766.7	-	-	-	-	-	-	-
4058	170	390	564	714	814	867	1108	1186	1274	2080
"	2505	2542	2674	-	-	-	-	-	-	-
4259	418	747	924	980	1512	1968	2062	2180	2483	3964
"	4062	4229	-	-	-	-	-	-	-	-

Ref. 20 Data Coding Number: 031 02 02 10 10

4068	53	61	67	67	70	71	73	76	79	90
4069	184	199	202	207	207	234	246	255	273	282
4070	457	498	585	592	608	641	663	667	802	11730
4274	1239	1345	1519	1574	1628	1883	1954	2602	3962	5847
4275	3497	3629	3695	3891	4282	4440	7045	17413	48779	57380

Ref. 73 Data Coding Number: 031 02 02 10 10

4277	879	879	923	963	1087	1090	1100	1196	1272	1367
4086	90	114	115	119	124	124	128	131	133	135
4085	87	96	100	104	105	105	106	112	115	115
"	121	121	122	123	124	126	128	130	139	157
4276	655	774	790	830	884	906	908	930	953	1004
"	1012	1041	1070	1128	1154	1155	1252	1259	1516	1664

Ref. 7 Data Coding Number: 031 11 02 10 10

1052	39.5	58.9	76.1	77.1	79.8	91.4	96.5	114.1	120.1	162.9
"	202.5	-	-	-	-	-	-	-	-	-

Ref. 9 Data Coding Number: 063 33 86 08 10

6000	34	35	36	47	48	48	52	63	77	78
"	80	85	92	97	105	114	117	118	125	130

Ref. 13 Data Coding Number: 090 10 85 50 80

410	97.4	99.2	114.4	115.1	115.6	115.8	115.8	121.4	123.0	125.7
"	140.9	142.1	-	-	-	-	-	-	-	-

TABLE 1. (CONTINUED)

Ref. 17 Data Coding Number: 197 02 86 50 10										
3205	122	145	153	161	163	163	184	187	194	194
"	195	199	202	204	205	212	212	212	216	216
3206	655	678	707	734	740	841	867	884	915	930
"	988	1000	1011	1018	1060	-	-	-	-	-

Ref. 17 Data Coding Number: 197 02 86 50 70										
3443	.093	.103	.103	.105	.107	.109	.113	.113	.122	.130
3444	.099	.103	.107	.110	.132	.136	.137	.140	.147	.152
3445	.076	.104	.111	.113	.115	.115	.129	.131	.134	.162
3446	.167	.177	.114	.117	.118	.120	.121	.124	.134	.162
3447	.100	.110	.133	.135	.140	.157	.163	.165	.165	.170
"	.180	.183	.135	.200	.201	.205	.215	.240	.253	.280

Ref. 20 Data Coding Number: 032 11 02 10 10										
1102	101	104	110	115	134	139	142	145	165	175
1103	145	187	225	248	260	281	283	284	286	382
1104	400	400	498	566	640	663	675	706	759	1236
1279	259	1197	1218	1658	1868	1889	2038	2455	2847	16533

Ref. 21 Data Coding Number: 032 02 06 00 10										
3062	519	598	614	661	731	807	840	857	943	1073
"	1285	-	-	-	-	-	-	-	-	-

Ref. 36 Data Coding Number: 040 02 81 20 10										
5784	40.3	42.3	42.6	44.0	48.3	47.6	47.9	55.8	66.0	102.2

Ref. 36 Data Coding Number: 040 02 81 20 11										
5783	34.7	35.0	37.8	39.7	42.3	47.6	47.9	55.8	66.0	.02.2

Ref. 63 Data Coding Number: 000 02 08 10 20										
5354	259	485	542	668	775	855	926	979	1053	1066
5483	1275	2085	2134	2212	2511	3511	3813	3862	4434	6496

Ref. 63 Data Coding Number: 000 02 08 10 21										
5353	190	219	282	483	500	694	697	752	770	814
5352	92	169	200	213	213	282	289	291	357	393
"	411	579	898	-	-	-	-	-	-	-
5481	233	271	538	704	725	764	1029	1132	1209	1225
"	1225	1573	1619	-	-	-	-	-	-	-

TABLE 1. (CONTINUED)

Ref. 72 Data Coding Number: 000 10 80 60 10

1200	.030	.035	.120	.138	.450	3.210	3.318	3.459	3.512	4.135
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Ref. 72 Data Coding Number: 000 01 80 60 10

4122	366	421	425	458	465	513	661	987	1157	1499
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Ref. 73 Data Coding Number: 031 02 02 10 60

4300	.067	.070	.087	.091	.095	.096	.102	.103	.124	.140
4301	.079	.095	.100	.104	.106	.111	.114	.117	.122	.197
4302	.074	.078	.081	.090	.097	.107	.113	.115	.119	.128
4304	.063	.067	.068	.073	.087	.094	.096	.096	.116	.124
4305	.074	.115	.118	.119	.126	.132	.140	.140	.142	.156
4306	.065	.072	.081	.088	.094	.096	.098	.115	.125	.133

Ref. 73 Data Coding Number: 031 02 02 10 70

4308	.096	.110	.114	.115	.117	.121	.130	.140	.145	.153
4310	.065	.067	.068	.071	.072	.073	.074	.094	.095	.140
4312	.080	.088	.089	.091	.095	.102	.109	.139	.143	.168
4314	.068	.078	.088	.092	.096	.096	.097	.110	.110	.111
4316	.181	.111	.118	.121	.121	.130	.133	.153	.161	.186
4309	.059	.059	.063	.070	.080	.092	.096	.104	.105	.105
"	.111	.115	.143	-	-	-	-	-	-	-

Ref. 73 Data Coding Number: 031 02 02 10 80

4317	.078	.082	.084	.085	.091	.096	.124	.132	.149	.158
4318	.087	.088	.114	.119	.119	.121	.128	.136	.153	.155
4319	.089	.121	.127	.147	.164	.174	.192	.201	.205	.208

TABLE 2. MAXIMUM LIKELIHOOD ESTIMATES AND TI TESTS

Item	MLE Program 6/73			OMLF Program 14/73		TI	Boeing		TI
	m	x_u	x_o	m	x_u	a><	nr	m	m \geq 1.0

Ref. 1 Data Coding Number: 040 10 86 00 50

459	1.287	2.4	0.8	1.193	228.0	602	10	6.298	800
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Ref. 3 Data Coding Number: 300 12 85 58 30

651	<1.0	-	-	.890	179.0	206	10	3.308	800
652	<1.0	-	-	.683	319.0	206	8	3.885	602
930	<1.0	-	-	.642	902.0	107	5	2.000	602
653	<1.0	-	-	.941	454.2	-	10	2.474	-

Ref. 3 Data Coding Number: 300 12 86 58 30

44	<1.0	-	-	.355	70.0	008	5	14.353	008
45	<1.0	-	-	.362	82.0	008	8	2.004	008
46	2.340	6.6	2.2	4.345	5.6	800	10	9.847	800
47	1.000	15.4	1.4	1.324	15.4	701	10	13.108	800
48	<1.0	-	-	.591	40.0	008	9	6.350	305
49	<1.0	-	-	.421	121.0	008	9	1.153	008
50	-	<0	-	-	<0	701	10	1.585	800
51	<1.0	-	-	-	<0	701	10	3.892	701
52	<1.0	-	-	.607	29.0	008	6	4.585	107
53	<1.0	-	-	-	<0	701	10	2.272	800
54	1.000	799.0	256.9	3.616	437.0	611	10	4.810	800
323	<1.0	-	-	.558	340.0	008	6	1.796	404
324	<1.0	-	-	1.188	10508.2	404	10	2.589	800
327	<1.0	-	-	1.099	242.0	305	10	1.236	800
328	<1.0	-	-	.452	1289.0	008	6	2.772	008
331	2.091	586.8	970.1	1.733	735.0	701	10	3.185	800
549	1.523	704.5	537.8	1.468	743.4	800	10	3.545	800
548	1.711	1515.9	457.8	1.621	1554.4	-	14	7.321	-
407	1.899	161.4	58.9	1.775	164.4	-	15	7.006	-

TABLE 2. (CONTINUED)

Item	MLE Program 6/73			OMLE Program 14/73		TI	Boeing		TI
	m	x _u	x _o	m	x _u	a><	nr	m	m _{≥1.0}
Ref. 4 Data Coding Number: 300 06 85 58 30									
3650	1.784	14.1	7.6	1.7912	15.0	800	10	5.2461	800
3651	-	<0	-	-	<0	710	10	6.6583	800
3653	-	<0	-	2.5585	334.0	800	6	5.7456	800
3654	<1.0	-	-	1.5959	558.5	800	10	3.1900	800
3655	3.640	32.6	814.0	2.3376	317.0	800	10	2.4843	800
3656	<1.0	-	-	1.4039	322.8	404	10	2.1432	800
3657	<1.0	-	-	.8220	606.0	206	8	4.5759	800
3658	<1.0	-	-	1.3762	78.5	404	5	12.5834	602
3659	<1.0	-	-	1.2185	217.0	701	10	2.8079	800
3660	2.065	110.5	384.5	-	<0	800	10	2.6689	800
3661	<1.0	-	-	.8623	210.0	404	7	5.9897	800
3662	<1.0	-	-	-	0	503	10	2.0829	800
3663	1.645	213.6	96.2	2.0096	214.7	800	10	5.7126	800
3922	<1.0	-	-	1.485	2244.0	800	10	4.3511	800
3923	<1.0	-	-	.754	12090.0	107	10	3.001	800
3927	<1.0	-	-	-	<0	620	10	1.753	800
3928	<1.0	-	-	1.851	12281.4	701	10	2.487	800
3929	1.540	364.2	2967.8	1.486	597.0	800	10	1.687	800
3930	<1.0	-	-	-	<0	620	10	5.298	800
3931	1.591	741.3	1509.9	1.514	876.0	800	10	2.425	800
3932	<1.0	-	-	.734	1190.0	305	6	3.016	800
3933	<1.0	-	-	1.339	2340.4	503	5	3.592	800
3934	<1.0	-	-	1.154	1526.0	602	10	2.400	800
3935	<1.0	-	-	1.520	1733.0	800	10	1.830	800
3936	-	<0	-	2.362	1924.0	800	10	2.877	800
3937	<1.0	-	-	-	<0	602	10	1.352	800
3938	1.595	413.9	1617.3	1.443	540.0	602	10	1.952	800
3939	<1.0	-	-	-	<0	314	10	2.076	800
3940	<1.0	-	-	.832	938.0	206	9	3.000	800
3652	<1.0	-	-	.822	377.0	-	6	5.746	-
3924	<1.0	-	-	1.164	19082.0	-	9	2.457	-
3926	<1.0	-	-	1.082	1158.0	-	11	2.064	-
3920	1.355	363.7	1196.1	1.095	393.0	206	19	2.913	-
3921	1.623	2092.3	5420.1	1.280	2292.0	5013	20	2.027	-

TABLE 2. (CONTINUED)

Item	MLE Program 6/73			OMLE Program 14/73		TI	Boeing		TI
	m	x _u	x _o	m	x _u	a><	nr	m	m≥1.0

Ref. 7 Data Coding Number: 031 02 02 10 10

4054	3.356	18.6	24.9	2.548	26.5	-	12	5.661	-
4055	2.839	9.3	54.7	-	<0	-	12	3.154	-
4056	1.545	40.1	72.8	1.552	42.5	-	12	2.510	-
4057	1.308	118.7	245.7	1.327	126.6	-	13	2.026	-
4058	1.336	122.3	1274.5	-	<0	-	13	1.472	-
4259	1.206	378.2	1849.3	-	<0	-	12	1.586	-

Ref. 20 Data Coding Number: 031 02 02 10 10

4068	2.198	45.0	28.8	2.079	53.0	800	10	7.092	800
4069	1.364	181.5	51.4	2.207	172.3	800	10	7.035	800
4070	1.000	457.0	211.6	1.084	457.0	305	10	3.103	800
4274	<10	-	-	.748	1239.0	206	7	6.384	800
4275	<10	-	-	.450	3497.0	107	6	10.156	107

Ref. 73 Data Coding Number: 031 02 02 10 10

4277	1.000	879.0	196.6	1.945	7967	701	10	4.440	701
4086	-	<0	-	-	<0	440	10	12.784	800
4085	2.485	80.3	41.0	-	-	17.0.1	20	7.047	18.0.0
4276	1.843	617.0	480.5	-	-	16.0.2	20	4.116	18.0.0

Ref. 7 Data Coding Number: 031 11 02 10 10

1052	1.476	35.1	73.3	1.408	39.5	-	11	2.214	-
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Ref. 9 Data Coding Number: 063 33 86 08 10

6000	1.001	34.0	45.1	-	-	14.0.4	20	2.632	18.0.0
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Ref. 13 Data Coding Number: 090 10 85 50 80

410	2.430	89.5	33.1	2.581	89.3	-	12	8.843	-
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TABLE 2. (CONTINUED)

Item	MLE Program 6/73			OMLE Program 14/73		TI	Boeing		TI
	m	x _u	x _o	m	x _u	a><	nr	m	m≥1.0

Ref. 17 Data Coding Number: 197 02 86 50 10

3205	-	<0	-	-	-	5.13.0	20	9.146	18.0.0
3206	4.933	322.4	597.4	-	<0	-	15	7.339	-

Ref. 17 Data Coding Number: 197 02 86 50 70

3443	2.213	0.1	0.02	1.840	0.09	800	10	10.066	800
3444	1.000	0.1	0.03	-	<0	800	10	7.491	800
3445	-	<0	-	8.605	0.03	620	10	7.822	800
3446	4.980	0.0	0.12	3.493	0.04	800	10	4.777	800
3447	2.270	0.1	0.11	-	-	18.0.0	20	4.028	18.0.0

Ref. 20 Data Coding Number: 032 11 10 10

1102	1.000	101.1	32.0	2.188	87.0	800	10	5.510	800
1103	3.090	86.1	192.1	2.171	145.0	800	10	4.217	800
1104	<1.0	-	-	1.271	351.0	503	10	2.668	701
1279	<1.0	-	-	0.743	259.0	107	9	2.268	503

Ref. 21 Data Coding Number: 032 02 06 00 10

3062	1.338	505.6	331.3	1.394	519.3	-	11	3.584	-
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Ref. 36 Data Coding Number: 040 02 81 20 10

5784	<1.0	-	-	0.776	40.3	107	10	2.784	800
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Ref. 36 Data Coding Number: 040 02 81 20 11

5783	<1.0	-	-	0.745	34.7	008	10	2.425	800
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Ref. 63 Data Coding Number: 000 02 08 10 20

5354	-	<0	-	-	<0	800	10	3.211	800
5483	1.391	1158.1	2264.1	1.414	1274.0	800	10	2.177	800

Ref. 63 Data Coding Number: 000 02 08 10 21

5353	-	<0	-	-	<0	710	10	2.428	800
5352	1.258	85.0	270.8	1.161	91.7	-	13	1.670	-
5481	-	<0	-	-	<0	-	13	2.178	-

TABLE 2. (CONTINUED)

Item	MLE Program 6/73			OMLE Program 14/73		TI	Boeing		TI
	m	x_u	x_o	m	x_u	a><	nr	m	$m \geq 1.0$

Ref. 72 Data Coding Number: 000 10 80 60 10

1200	<1.0	-	-	-	<0	314	5	0.869	602
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Ref. 72 Data Coding Number: 000 01 80 60 10

4122	<1.0	-	-	.893	366.0	305	6	9.245	800
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Ref. 73 Data Coding Number: 031 02 02 10 60

4300	1.604	0.06	0.04	1.751	0.06	800	10	4.440	800
4301	1.248	0.08	0.04	1.196	0.08	404	10	3.288	800
4302	1.000	0.07	0.03	5.721	0.02	800	10	5.903	800
4304	1.000	0.06	0.03	1.819	0.06	800	10	4.394	800
4305	-	<0	-	4.114	0.07	620	10	7.123	800
4306	1.761	0.06	0.04	2.018	0.06	800	10	4.586	800

Ref. 73 Data Coding Number: 031 02 02 10 70

4308	2.309	0.09	0.04	2.133	0.10	800	10	7.318	800
4310	<1.0	-	-	0.746	0.06	107	10	3.268	701
4312	1.000	0.08	0.03	1.192	0.08	404	10	3.723	800
4314	-	<0	-	4.678	0.05	800	10	8.309	800
4316	2.655	0.06	0.08	2.074	0.08	800	10	4.629	800
4309	1.000	0.06	0.03	2.211	0.04	-	13	3.860	-

Ref. 73 Data Coding Number: 031 02 02 10 80

4317	<1.0	-	-	1.356	0.08	503	10	3.692	800
4318	3.557	0.05	0.08	4.042	0.05	800	10	5.847	800
4319	-	<0	-	-	<0	530	10	4.708	800

TABLE 3. DATA CODING SYSTEM

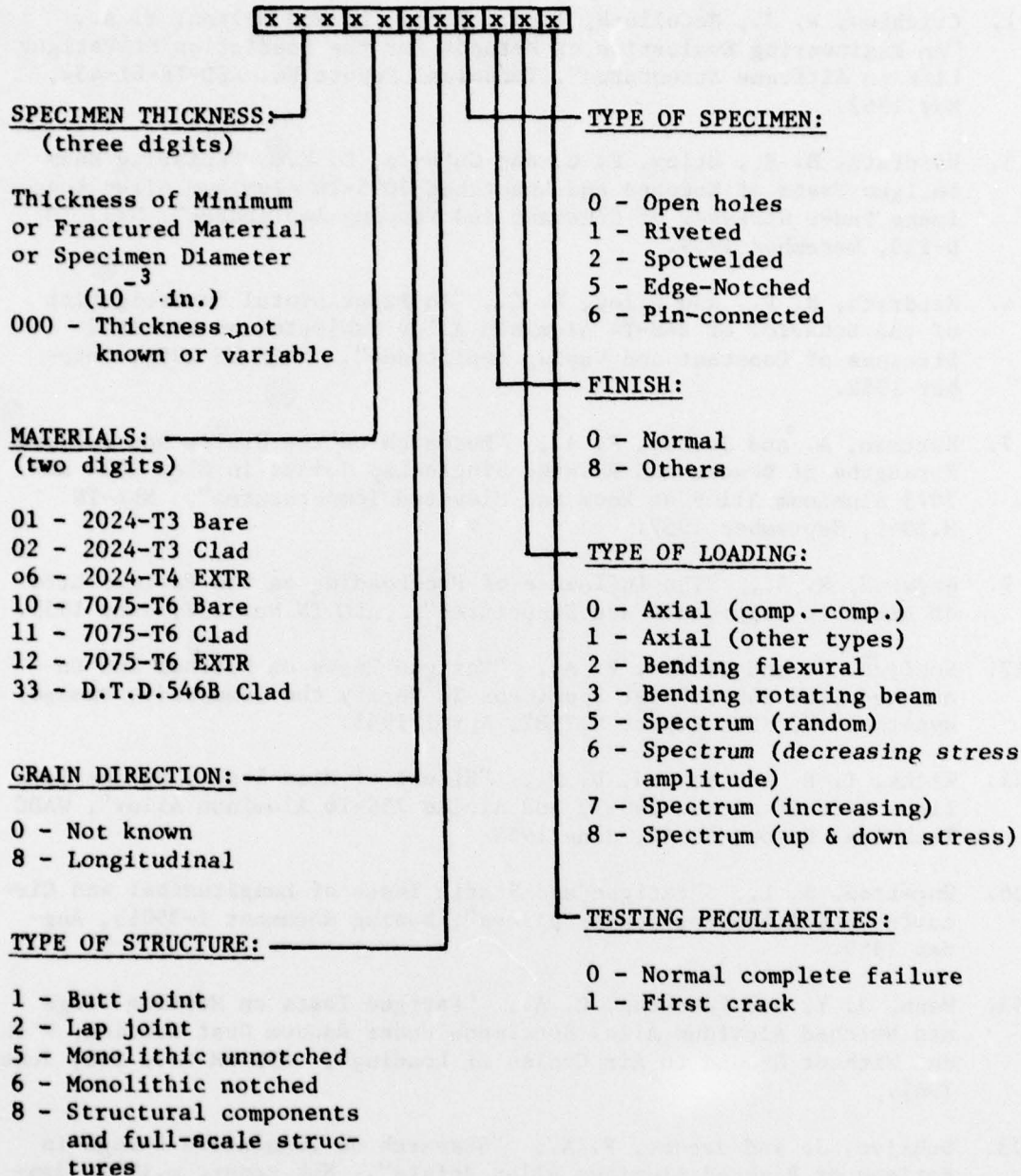


TABLE 4. REFERENCES TO THE BOEING ITEMS

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3. Hardrath, H. F., Utley, E. C. and Guthrie, D. E., "Rotating Beam Fatigue Tests of Notched and Unnotched 7075-T6 Aluminum Alloy Specimens Under Stresses of Constant and Varying Amplitudes", NASA TN D-210, December 1959.
4. Hardrath, H. F., and Utley, E. C., "An Experimental Investigation of the Behavior of 24S-T4 Aluminum Alloy Subjected to Repeated Stresses of Constant and Varying Amplitudes", NACA TN 2798, October 1952.
7. Hartman, A. and Jacobs, F. A., "Research on the Static and Fatigue Strengths of Bonded and Riveted Single-Lap Joints in Clad 2024 and 7075 Aluminum Alloy at Room and Elevated Temperatures", NLL-TN M.2041, September 1957.
9. Heywood, R. B., "The Influence of Pre-Loading on the Fatigue Life of Aircraft Components and Structures", ARC TN No. 232, June 1955.
17. Schijve, J. and Jacobs, F. A., "Fatigue Tests on Notched and Unnotched Clad 24S-T Sheet Specimens To Verify the Cumulative Damage Hypotheses", NLL Report M.1982, April 1955.
21. Wilks, I. E. and Howard, D. M., "Effect of Mean Stress on the Fatigue Life of Alclad 24S-T3 and Alclad 75S-T6 Aluminum Alloy", WADC Technical Report 53-40, June 1953.
36. Engstrom, S. L., "Fatigue and Static Tests of Longitudinal and Circumferential Fuselage Skin Splices", Boeing document T-29015, August 1955.
63. Mann, J. Y. and Patching, C. A., "Fatigue Tests on Mustang Wings and Notched Aluminum Alloy Specimens Under Random Gust Loading, With and Without Ground to Air Cycles of Loading", ARL SM Note 268, June 1961.
73. Schijve, J. and Jacobs, F. A., "Research on Cumulative Damage in Fatigue of Riveted Aluminum Alloy Joints", NLL Report m.1999, January 1956.

TABLE 4. (CONTINUED)

13. Naumann, C., "Evaluation of the Influence of Load Randomization and of Ground-Air-Ground Cycles on Fatigue Life", NASA TN D-1584, October 1964.
20. Schijve, J. and Jacobs, F. A., "Program-Fatigue Tests on Notched Light Alloy Specimens of 2024 and 7075 Material", NLL Technical Report M.2070, 1960.
72. Smith, C. R., "Linear Strain Theory and the Smith Method for Predicting Fatigue Life of Structures for Spectrum-Type Loading", ARL 64-55, April 1964.

TABLE 5. REJECTION OF THE HYPOTHESIS H_0

Item	MLE	OMLE	TI
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DCN:	040	10	86	00	50
459	No	No	No	No	No

DCN:	300	12	85	58	30
651	Yes	Yes	No	No	No
652	Yes	Yes	Yes	Yes	Yes
930	Yes	Yes	Yes	Yes	Yes
653	Yes	Yes	Yes	-	-

DCN:	300	12	85	58	30
44	Yes	Yes	Yes	Yes	Yes
45	Yes	Yes	Yes	Yes	Yes
46	No	No	No	No	No
47	No	No	No	No	No
48	Yes	Yes	Yes	Yes	Yes
49	Yes	Yes	Yes	Yes	Yes
50	Yes	Yes	No	No	No
51	Yes	Yes	Yes	Yes	Yes
52	Yes	Yes	Yes	Yes	Yes
53	Yes	Yes	No	No	No
54	No	No	No	No	No
323	Yes	Yes	Yes	Yes	Yes
324	Yes	No	No	No	No
327	Yes	No	No	No	No
328	Yes	Yes	Yes	Yes	Yes
331	No	No	No	No	No
549	No	No	No	No	No
548	No	No	-	-	-
407	No	No	-	-	-

DCN:	031	02	02	10	10
4054	No	No	-	-	-
4055	No	Yes	-	-	-
4056	No	No	-	-	-
4057	No	No	-	-	-
4058	No	Yes	-	-	-
4059	No	Yes	-	-	-

Item	MLE	OMLE	TI
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DCN:	300	06	85	59	30
3650	No	No	No	No	No
3651	Yes	Yes	No	No	No
3653	Yes	No	No	No	No
3654	Yes	No	No	No	No
3655	No	No	No	No	No
3656	Yes	No	No	No	No
3657	Yes	Yes	No	No	No
3658	Yes	No	Yes	Yes	Yes
3659	Yes	No	No	No	No
3660	No	Yes	No	No	No
3661	Yes	Yes	No	No	No
3662	Yes	Yes	No	No	No
3663	No	No	No	No	No
3922	Yes	No	No	No	No
3923	Yes	Yes	No	No	No
3927	Yes	Yes	No	No	No
3929	No	No	No	No	No
3930	Yes	Yes	No	No	No
3931	No	No	No	No	No
3932	Yes	Yes	No	No	No
3933	Yes	No	No	No	No
3934	Yes	No	No	No	No
3935	Yes	No	No	No	No
3936	Yes	No	No	No	No
3937	Yes	Yes	No	No	No
3938	No	No	No	No	No
3939	Yes	Yes	No	No	No
3940	Yes	Yes	No	No	No
3652	Yes	Yes	No	No	No
3924	Yes	No	No	No	No
3926	Yes	No	No	No	No
3920	No	No	No	No	No
3921	No	No	No	No	No
3928	Yes	No	No	No	No

TABLE 5. (CONTINUED)

Item	MLE	OMLE	TI
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DCN: 031 02 02 10 10

4068	No	No	No
4069	No	No	No
4070	No	No	No
4274	Yes	Yes	No
4275	Yes	Yes	Yes

DCN: 063 33 86 08 10

6000	No	-	No
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DCN: 090 10 85 50 80

410	No	No	-
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DCN: 197 02 86 50 10

3205	Yes	-	No
3206	No	Yes	-

DCN: 197 02 86 50 70

3443	No	No	No
3444	No	Yes	No
3445	Yes	No	No
3446	No	No	No
3447	No	-	No

DCN: 032 11 02 10 10

1102	No	No	No
1103	No	No	No
1104	Yes	No	Yes
1279	Yes	Yes	Yes

DCN: 032 02 06 00 10

3062	No	No	-
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DCN: 040 02 81 20 10

5784	Yes	Yes	No
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DCN: 040 02 81 20 11

5783	Yes	Yes	No
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DCN: 000 02 08 10 20

5354	Yes	Yes	No
5483	No	No	No

Item	MLE	OMLE	TI
------	-----	------	----

DCN: 031 02 02 10 10

4277	No	No	Yes
4086	Yes	Yes	No
4085	No	-	No
4276	No	-	No

DCN: 031 11 02 10 10

1052	No	No	-
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DCN: 000 02 08 10 21

5353	Yes	Yes	No
5352	No	No	-
5481	Yes	Yes	-

DCN: 000 10 80 60 10

1200	Yes	Yes	Yes
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DCN: 000 01 80 60 10

4122	Yes	Yes	No
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DCN: 031 02 02 10 60

4300	No	No	No
4301	No	No	No
4302	No	No	No
4304	No	No	No
4305	Yes	No	No
4306	No	No	No

DCN: 031 02 02 10 70

4308	No	No	No
4310	Yes	Yes	Yes
4312	No	No	No
4314	Yes	No	No
4316	No	No	No
4309	No	No	-

DCN: 031 02 02 10 80

4317	Yes	No	No
4318	No	No	No
4319	Yes	Yes	No